

Simplified Model for Ionization and Recombination in a Hydrogenic Plasma with Resonance Radiation Trapping

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(Z. Naturforsch. **25 a**, 1798—1803 [1970]; received 24 August 1970)

The rate coefficients for electron collisional-radiative ionization and recombination have been calculated on the basis of a simplified level scheme for hydrogenic ions in a homogeneous and fully contained plasma with self-absorption for (Doppler broadened) Lyman α radiation. The energy distribution of the electrons is assumed to be Maxwellian. Photoionization is neglected. The calculation have been done for a range of electron temperature of Z^2 to $10 Z^2$ eV and electron density of $10^{14} Z^7$ to $10^{24} Z^7 \text{ m}^{-3}$, where Z is the charge number of the bare nucleus.

1. Introduction

For a hydrogenic plasma, i.e., a plasma that consists of ions or atoms with only one electron around the nucleus and of bare nuclei, the electronic ionization and recombination rate coefficients have been numerically computed in detail. The plasma was assumed to be optically thin, either throughout the spectrum (case A)^{1,2,3}, or throughout the spectrum except for resonance radiation, which is completely absorbed (case B)^{3,4}.

However, if the plasma is neither optically thin nor thick towards the resonance lines the rate coefficients depend also on the optical depths of the plasma for the resonance lines. An exact computation is complicated and would require a detailed solution of the equations for radiative transfer coupled with those determining the level populations of the ions or atoms.

In this paper we have tried, for the purpose of making estimates in plasma physical problems, to extend the results from previous computer calculations¹⁻⁴ to the intermediate range between cases A and B by using an approximate model⁵ for the collisional and radiative processes in the hydrogenic ion or atom.

According to Holstein's treatment⁶ the influence of resonance radiation trapping is taken into account by the reduction of the spontaneous transition prob-

ability of the Lyman α line by a numerical factor indicating the average probability for the resonance photons to escape out of the plasma volume.

In the above-mentioned computations the assumption is always made that for each set of values of the electron temperature, T_e , and density, n_e , respectively, the population density of each excited state of the ion or atom is established instantaneously, without the number densities of free electrons, bare nuclei, and hydrogenic ions (atoms) being appreciably altered. Such a quasi-equilibrium, in which the excited level populations can be referred to a particular set of densities of ions (atoms) in the ground state, free electrons, and bare nuclei, is possible if the relaxation times for the excited levels are much shorter than those for the ground level or the free electrons. It can be shown that in the considered region of electron temperatures and densities the conditions for the quasi-equilibrium are satisfied⁴. For the results to be applicable to transient plasmas, it has to be verified that the relaxation times for the equilibrium are much shorter than the characteristic times for the relevant plasma parameters, e.g., T_e and n_e .

In the computations it is assumed that the electrons have a Maxwellian energy distribution. This implies that the mean free path for elastic electron-electron collisions must be smaller than the plasma dimensions and, in the case of a transient plasma,

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¹ D. R. BATES, A. E. KINGSTON, and R. W. P. McWHIRTER, *Proc. Roy. Soc. London A* **267**, 297 [1962].

² H.-W. DRAWIN, *Ann. Phys. Leipzig* **14**, 262 [1964].

³ H.-W. DRAWIN, *Z. Phys.* **225**, 470 [1969] and Report EUR-CEA-FC-510, Association EURATOM-CEA, Fontenay-aux-Roses 1969.

⁴ D. R. BATES, A. E. KINGSTON, and R. W. P. McWHIRTER, *Proc. Roy. Soc. London A* **270**, 155 [1962].

⁵ R. MEWE, *Brit. J. Appl. Phys.* **18**, 107 [1967].

⁶ T. HOLSTEIN, *Phys. Rev.* **72**, 1212 [1947]; **83**, 1159 [1951].



the electron-electron collision time must be much shorter than the characteristic times of the plasma parameters. Further, the ratio of the electron to the hydrogenic ion density must be high enough (i.e., $\gg 10^{-2}$ in the considered electron temperature region) in order that inelastic collisions between electrons and ions cannot influence the tail of the electron energy distribution.

For example, in a pinch discharge of modest temperature, say several tens of eV, the conditions above are met approximately.

Because in the equations of continuity the divergences of the particle fluxes are ignored (assumption of homogeneity), the results are not applicable to those regions of the plasma where strong gradients in temperature and density exist.

The plasma is assumed to be fully contained in that sense that diffusion into and from the plasma particles, especially neutrals⁷, cannot affect the degree of ionization and excitation in the plasma. For a hydrogen plasma the effect of atom-atom collisions⁸ is neglected so that the ionization degree should be high enough (i.e., $\gg 10^{-3}$).

Finally, photoionization is neglected, i.e., the plasma is assumed to be optically thin towards continuum radiation.

2. Scheme of Processes and Rate Equations

The physical processes, which occur in the ionization of hydrogenic ions or atoms of charge number $Z-1$ to form bare nuclei of charge number Z as well as in the inverse recombination process, will be considered on the basis of a simplified level scheme. These processes, with their rate coefficients in parenthesis, are (cf. Fig. 1):

(i) Collisional ionization from level 1 (S_{1c}), (ii) radiative recombination to level 1 (α_{c1}), (iii) collisional excitation $1 \rightarrow 2$ (S_{12}), (iv) collisional de-excitation $2 \rightarrow 1$ (S_{21}), (v) radiative decay (corrected for self-absorption) of level 2 to the ground level 1 ($T_{21}A_{21}$), (vi) collisional ionization from level 2, direct and, at higher n_e , also through excitation from upper bound levels p (S_{2c}), (vii) collisional-radiative recombination to level 2, i.e., radiative and three-body recombination, either direct or stepwise through radiative decay and collisional de-excitation from levels p (α_{c2}).

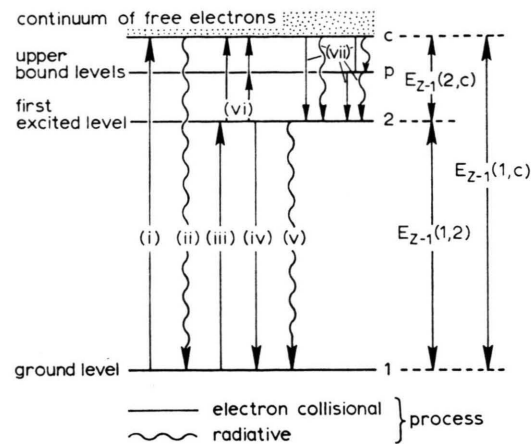


Fig. 1. Schematic representation of electron collisional and radiative processes in a simplified level diagram for a hydrogenic ion or atom with charge number $Z-1$.

Dielectronic recombination⁹ is not possible because for this process the ion must have at least two electrons.

In the level diagram

$$E_{Z-1}(p, q) = 13.6 Z^2 |p^{-2} - q^{-2}| \text{ (eV)}$$

is the absolute energy difference between the levels with principal quantum numbers p and q , respectively, and $E_{Z-1}(p, c) = 13.6 Z^2 p^{-2}$ (eV) the energy of ionization from level p to the continuum (c).

In the hydrogenic level scheme the first excited level lies already at $3/4$ times the total ionization energy above the ground level. Because we will assume that $kT_e < E_{Z-1}(1, 2)$ we may expect that the first excited level will play a dominant role in the stepwise processes. Therefore, in the rate equations for the level populations we have presented explicitly only the ground and first excited level and we have put the effect of the higher levels into the rate coefficients S_{2c} and α_{c2} , which therefore depend, except on T_e , also on n_e .

We now consider a plasma that consists of $Z-1$ times charged hydrogenic ions (atoms) and of Z times charged bare nuclei. It is assumed here that the preceding ionization stage is stationary so that ionization or recombination of ions of charge $Z-2$ or $Z-1$, respectively, can be neglected. The same holds for the succeeding ionization stage ($Z \rightleftharpoons Z+1$). Then the rate equations that govern

⁷ H.-W. DRAWIN, Z. Phys. **186**, 99 [1965].

⁸ H.-W. DRAWIN, Z. Phys. **225**, 483 [1969].

⁹ A. BURGESS, Astrophys. J. **139**, 776 [1964].

the population densities, $n_{Z-1}(1)$ and $n_{Z-1}(2)$, of the ground and first excited level, respectively, of the hydrogenic ion or atom can be written as:

$$\partial n_{Z-1}(1)/\partial t = -n_{Z-1}(1) n_e (S_{1c} + S_{12}) + n_e \{n_{Z-1}(2) (S_{21} + T_{21} A_{21}/n_e) + n_Z \alpha_{c1}\}, \quad (1)$$

$$\partial n_{Z-1}(2)/\partial t = -n_{Z-1}(2) n_e (S_{2c} + S_{21} + T_{21} A_{21}/n_e) + n_e \{n_{Z-1}(1) S_{12} + n_Z \alpha_{c2}\}, \quad (2)$$

where n_Z is the number density of the bare nuclei. The divergences of the particle fluxes are neglected in the equations because the plasma is assumed to be homogeneous and fully contained. Provided the relaxation time of the excited level is much shorter than that of the ground level, which is generally true^{10, 11}, it follows that $n_{Z-1}(2)$ quickly rises to a quasi-equilibrium value (the symbol Q denoting the quasi-equilibrium)

$$n_{Z-1, Q}(2) = \frac{n_{Z-1}(1) S_{12} + n_Z \alpha_{c2}}{S_{2c} + S_{21} + T_{21} A_{21}/n_e}$$

and that thereafter

$$\partial n_{Z-1}(1)/\partial t = -n_{Z-1}(1) n_e S_{Z-1} + n_Z n_e \alpha_Z, \quad (3)$$

where

$$S_{Z-1} = S_{1c} + \frac{S_{2c}}{S_{2c} + S_{21} + T_{21} A_{21}/n_e} S_{12} \quad (4)$$

and

$$\alpha_Z = \alpha_{c1} + \frac{S_{21} + T_{21} A_{21}/n_e}{S_{2c} + S_{21} + T_{21} A_{21}/n_e} \alpha_{c2}. \quad (5)$$

Because, if $k T_e < E_{Z-1}(1, 2)$, the excited level populations are much less than the ground level population, and the quasi-equilibrium ensures that the population of the excited levels is established effectively instantaneously, it follows that

$$\partial n_{Z-1}(1)/\partial t \cong \partial n_{Z-1}/\partial t,$$

where n_{Z-1} is the total density of ions (atoms) (including all excited states) with charge number $Z-1$. The fraction in the right-hand member of Eq. (5) or (6) represents the relative number of electrons transferred out of level 2 that goes upward or downward, respectively. The quantities S_{Z-1} and α_Z are to be interpreted as effective coefficients for ionization and recombination, resulting from the combined action of collisional and radiative processes. They were named by BATES et al.^{1, 4} collisional-

radiative ionization and recombination coefficient, respectively. These coefficients depend on T_e , n_e , and various atomic parameters.

According to appropriate scaling laws¹ we can write the temperatures, densities, and rate coefficients in a reduced form that becomes independent of the specific nuclear charge number, Z . We introduce the reduced electron temperature and density

$$\Theta = k T_e / Z^2, \quad \eta = n_e / Z^7,$$

and the reduced rate coefficients

$$S_{pq}^* = Z^3 S_{pq}, \quad S^* = Z^3 S_{Z-1}, \\ \alpha_{pc}^* = \alpha_{pc} / Z, \quad \alpha^* = \alpha_Z / Z, \quad A_{pq}^* = A_{pq} / Z^4.$$

Substituting the reduced quantities into the Eqs. (4) and (5) we obtain for the reduced coefficients for ionization and recombination

$$S^* = S_{1c}^* + \frac{S_{2c}^*}{S_{2c}^* + S_{21}^* + T_{21} A_{21}^* / \eta} S_{12}^*, \quad (6)$$

$$\alpha^* = \alpha_{c1}^* + \frac{S_{21}^* + T_{21} A_{21}^* / \eta}{S_{2c}^* + S_{21}^* + T_{21} A_{21}^* / \eta} \alpha_{c2}^*. \quad (7)$$

be for the region $\Theta = 1$ to 10 eV (where the change of ionization phase usually takes place)¹²:

$$S_{1c}^* = 9.4 \times 10^{-15} A \Theta^{1/2} \exp\{-13.6/\Theta\}, \quad (8a)$$

$$S_{2c}^* = 2.1 \times 10^{-13} \{1 + 30 \Theta^{-1/2} \eta^{1/2} / (\eta + 10^{20})^{1/2}\} \\ \cdot \exp\{-3.4/\Theta\}, \quad (8b)$$

$$S_{12}^* = 1.2 \times 10^{-13} B \Theta^{-1/2} \exp\{-10.2/\Theta\}, \quad (8c)$$

$$S_{21}^* = 3.0 \times 10^{-14} B \Theta^{-1/2}, \quad (8d)$$

$$\alpha_{c1}^* = 1.2 \times 10^{-19} \Theta^{-1/2}, \quad (8e)$$

$$\alpha_{c2}^* = 2.5 \times 10^{-19} \Theta^{-1} \\ \cdot \{1 + 3 \times 10^{-20} \Theta^{-1} \eta (\eta + 10^{20}) / (\eta + 10^{18})\}, \quad (8f)$$

$$A_{21}^* = 4.7 \times 10^8, \quad (8g)$$

where the collisional rate coefficients are expressed in $\text{m}^3 \text{s}^{-1}$, the transition probability in s^{-1} , Θ in eV, and η in m^{-3} . The values of the coefficients A and B are: $A = 0.6 (Z=1)$, $1 (Z=2)$, $1.3 (Z>2)$; $B = 0.04 \Theta (Z=1)$, $1 (Z>1)$. In the expressions for S_{2c}^* and α_{c2}^* the factors between braces represent the effect of stepwise processes through the upper bound levels. They have been fitted empirically by a comparison with Drawin's³ computations.

¹⁰ R. W. P. MCWHIRTER and A. G. HEARN, Proc. Phys. Soc. London **82**, 641 [1963].

¹¹ H.-W. DRAWIN, J. Quant. Spectrosc. Radiat. Transfer **10**, 33 [1970].

¹² R. MEWE, Rijnhuizen Report 70-61 (in English), Jutphaas 1970.

3. Trapping of Resonance Radiation

The effect of resonance radiation trapping we take into account by reducing the spontaneous emission probability, A_{21} , of the Lyman α transition $2 \rightarrow 1$ by a fractional factor T_{21} , introduced by Holstein who named it escape factor⁶. It represents the average probability for escape from the plasma volume of a resonance photon emitted in the transition $2 \rightarrow 1$. For a Doppler broadened line it has, for large optical thickness, the following asymptotical form⁶:

$$T_{21} \cong C/\tau (\ln \tau)^{1/2} \quad (\tau \gg 1), \quad (9)$$

where τ is the total optical thickness of the plasma at the line centre, and C is a numerical factor of the order of 1. This formula is valid under the conditions that the rate of de-excitation by electron collisions of the resonance level is less than the net radiative de-excitation rate: $n_e S_{21} \ll T_{21} A_{21}$. In the high electron density limit ($n_e S_{21} \gg T_{21} A_{21}$) C begins to deviate from one and approaches about the value $\ln \tau$ ¹³. However, in this case the precise value of T_{21} does not matter because then the population of the upper level 2 is determined by collision processes.

Since a comparison¹⁴ between the escape factor method and detailed numerical calculations of the radiative transfer in a homogeneous plasma slab with two-level atoms¹⁵ shows that the results for the excited level populations in the centre of the plasma always agree within 50% for $\tau = 10^{-1} - 10^4$, it seems to be justified to estimate the influence of radiation trapping in such an approximative manner.

We have slightly modified Eq. (9) (with $C = 1$) so as to extend its validity towards small optical depths. By writing

$$T_{21} = [1 + \tau \{\ln(1 + \tau)\}^{1/2}]^{-1} \quad (10)$$

we obtain a formula that fits within 30% the expression given by MITCHELL and ZEMANSKY¹⁶ for $\tau \leq 4.5$ (the quantity A_1 on p. 323 of the quoted reference).

The optical depth, τ , equals the absorption coefficient per unit length at the line centre times a

characteristic linear dimension, l , of the homogeneous plasma volume. Taking $C = 1$ in Eq. (9) we find for several geometries:

infinite plasma slab:

$$l = L = \text{the thickness of the slab}^6;$$

infinitely long plasma cylinder:

$$l = R = \text{the radius of the cylinder};$$

sphere:

$$l = \frac{1}{2} R = \text{half the radius of the sphere.}$$

The optical depth at the centre of a Doppler broadened Lyman α line of a homogeneous plasma cylinder of radius R is given by¹⁷

$$\tau = 5.44 \times 10^{-18} R^* \eta \Theta^{-1/2}, \quad (11)$$

where the reduced radius, R^* , is given by

$$R^* = Z^4 (n_{Z-1}(1)/n_e) (M_{Z-1} T_e / T_{Z-1})^{1/2} R. \quad (12)$$

Here, T_{Z-1} is the temperature of the hydrogenic ions (atoms) of charge $Z-1$, and M_{Z-1} is the ion mass in terms of the mass of a hydrogen atom. The latter formula shows that the effect of self-absorption on the reduced ionization and recombination rates rapidly increases with increasing atomic number, Z .

We will consider two important assumptions made in this section.

(i) Stimulated emission is negligible compared with spontaneous emission. If the Lyman α intensity (of frequency ν) would be equal to the black-body intensity corresponding to the electron temperature, the ratio of spontaneous and induced emission is $\exp\{h\nu/kT_e\} - 1$ ^{14, 15, 5}. Thus, if $kT_e < h\nu$ or $\Theta < 10.2$, the assumption is practically always valid.

(ii) The resonance line profile is determined only by Doppler broadening. We will briefly consider the effect that some other broadening mechanisms may possibly have.

Natural broadening is negligible compared with Doppler broadening if¹⁵ $\tau < 10^7 (kT_{Z-1}/Z^4 M_{Z-1})$, where the ion temperature, kT_{Z-1} , is expressed in eV.

Pressure (Stark) broadening can be ignored if¹⁵

$$\tau < 4.6 \times 10^{36} (kT_{Z-1}/Z^4 M_{Z-1} \eta^{4/3})^{5/4},$$

¹³ P. J. WALSH, Phys. Rev. **107**, 338 [1957].

¹⁴ R. W. P. MCWHIRTER, Ch. 5 of Plasma Diagnostic Techniques (Eds. R. H. HUDDLESTONE and S. L. LEONARD), Academic Press, New York 1965.

¹⁵ A. G. HEARN, Proc Phys. Soc. **81**, 648 [1963].

¹⁶ A. C. G. MITCHELL and M. W. ZEMANSKY, Resonance Radiation and Excited Atoms, The University Press, Cambridge 1934.

¹⁷ C. W. ALLEN, Astrophysical Quantities, 2nd Ed., The Athlone Press, London 1963.

where η is expressed in m^{-3} . With neutral atoms ($Z=1$) also resonance broadening¹⁸ can occur. It is negligible if^{18, 15} $\tau < 10^{51} [k T_0 / M_0 n_0^2(1)]$, where $n_0(1)$ is the density of neutral atoms in the ground state (in m^{-3}) and $k T_0$ is the atom temperature (in eV).

Finally, the Zeeman splitting of a normal Lorentz triplet¹⁹ in a magnetic field, B , is smaller than the full Doppler half-width if $B < 6.7 (k T_{Z-1} Z^4 / M_{Z-1})^{1/2}$ (Tesla).

4. Numerical Results for S^* and α^* and Discussion

From Eqs. (6) – (8) and (10) – (12) one can evaluate the coefficients S^* and α^* . As an example, we have plotted in Figures 2 to 4 the results against η for $\Theta=1, 3$, and 10 eV and $Z=2$ (helium).

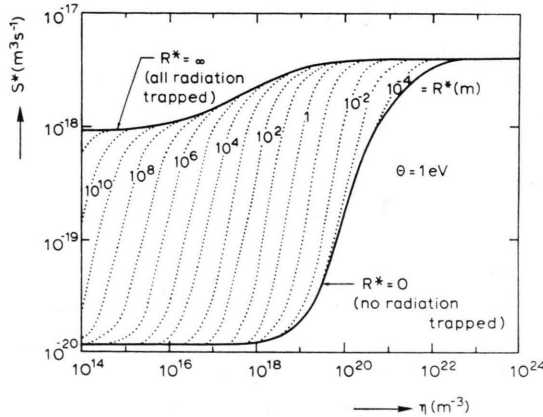


Fig. 2. Reduced ionization coefficient as a function of reduced electron density ($Z=2$).

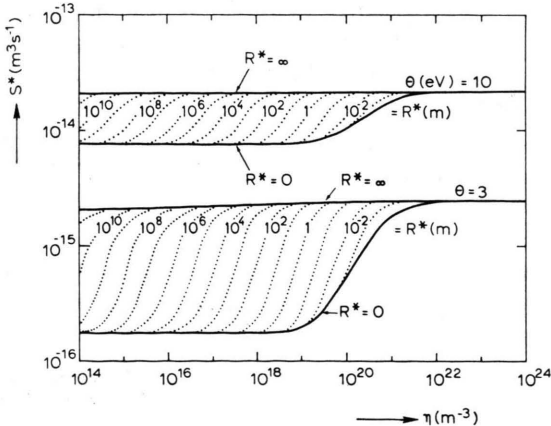


Fig. 3. Reduced ionization coefficient as a function of reduced electron density ($Z=2$).

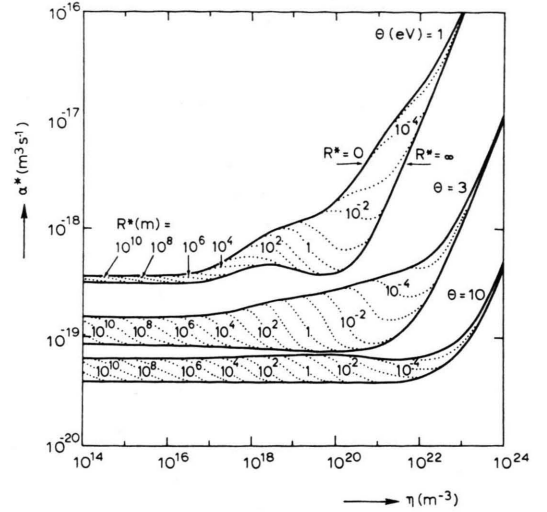


Fig. 4. Reduced recombination coefficient as a function of reduced electron density ($Z=2$).

The reduced radius, R^* , represents the effect of resonance radiation trapping. We will estimate it for a plasma consisting of ions and atoms of only one species [with atomic number Z and total density $n(Z)$] and being in the ionization stage $(Z-1) \rightleftharpoons (Z)$. Hence we may assume

$$n_{Z-1}(1) \approx n_{Z-1} \approx n_Z \approx \frac{1}{2} n(Z)$$

and $n_e \approx (Z - \frac{1}{2}) n(Z)$. Assuming $M_{Z-1} \approx 2Z$ and $T_e \approx T_{Z-1}$ we find from Eq. (12)

$$R^* \approx Z^4 (2Z)^{1/2} (2Z-1)^{-1} R \approx Z^{7/2} R. \quad (13)$$

If also other kinds of ions (of charge X and density n_X) are present in the plasma, R^* must be reduced by a factor

$$1 / \left\{ 1 + \left(\sum_X n_X X \right) / (Z - \frac{1}{2}) n(Z) \right\}.$$

In a transient plasma in the stage $(Z-1) \rightleftharpoons (Z)$ self-absorption is more important than in a steady-state plasma because in the latter case the density of the hydrogenic ions (atoms) is generally much lower. For example, in a transient helium plasma ($Z=2$) with $\Theta=3$ eV and $R=0.01$ m the transition between the cases of optical thin and thick plasma occurs at $\eta=10^{19}-10^{20} \text{ m}^{-3}$ [cf. Figs. 3 and 4, and Eq. (13)] and in the steady-state case⁵ only at $\eta=10^{21}-10^{22} \text{ m}^{-3}$.

We may note that the effect of resonance radiation trapping on the ionization rate has also been

¹⁸ A. UNSÖLD, Physik der Sternatmosphären, Springer-Verlag, Berlin 1955.

¹⁹ H. E. WHITE, Introduction to Atomic Spectra, McGraw-Hill, New York 1934.

calculated for a cesium plasma on the basis of a similar model²⁰. The transition curves show a course that is comparable with the one given in Figures 2 and 3.

Finally, comparing our results with the solutions of numerical calculations³ we find in the limiting cases of optical thin and thick plasmas an agreement within about 50%. The transition curves between the two limiting cases may have a greater inaccuracy, say, of about a factor of two.

The author is grateful to Professor C. M. BRAAMS for his stimulating interest and to Dr. A. J. POSTMA for critically reading the manuscript.

This work was performed as part of the research programme of the association agreement of Euratom and the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM) with financial support from the "Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek" (ZWO) and Euratom.

²⁰ A. J. POSTMA, Rijnhuizen Report 66-36 (in English), Jutphaas 1966.

On the Heating of a Pinch

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(Z. Naturforsch. 25 a, 1803—1807 [1970]; received 24 August 1970)

The compression temperature of a theta pinch is calculated as a function of the circuit parameters and the final β -value of the plasma. One of the results is that the temperature, \hat{T} , at the peak magnetic field, \hat{B} , scales of $(\dot{B}\hat{B})^{1/2}$, where \dot{B} is the initial rate of rise of the magnetic field. A possibility of combining two capacitor banks to increase the implosion heating rate is discussed.

1. Introduction

In the course of work on screw pinches at Jutphaas it has been shown¹⁻⁵ that for a toroidal screw pinch with a longitudinal current of the order of the Kruskal-Shafranov limit to be stable against kink modes the β of the plasma must be kept low, say, $\beta \lesssim 0.25$ (β = kinetic plasma pressure/confining magnetic field pressure). In such a screw pinch the ratio of the axial and azimuthal magnetic field is of the order of R/r_1 , in which R is the major and r_1 is the minor radius of the toroidal discharge tube¹⁻⁴. As in practice $R/r_1 \gg 1$, the plasma compression is mainly determined by the axial field, so that the heating mechanism is similar to that in a common theta pinch.

The theta pinch is well known for its capability to produce a hot and dense plasma with a high β . However, if the β is lowered by trapping a magnetic

bias field parallel to the main confining field, the heating becomes less effective because of the reduction of the compression ratio⁶⁻⁸. The production of a low- β pinch — such as the above-mentioned screw pinch — therefore requires a large capacitor bank. This paper gives the formulae necessary to calculate the parameters of this bank. A reduction of costs is possible if, in addition to the main capacitor bank, a small fast bank is used. This method, described earlier by DE VRIES³, has the advantage of a high initial field rise and therefore of an increased heating during the fast implosion.

2. Formulae for the Heating of a Theta Pinch

In a theta pinch device the plasma is produced and heated by the shock and adiabatic compression due to a fast rising magnetic field induced by discharging a low-inductance capacitor bank through a coil around the discharge tube.

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¹ C. BOBELDIJK, Rijnhuizen Report 68-45, Thesis, Utrecht 1968.

² C. BOBELDIJK et al., Proc. 3rd Conf. on Plasma Phys. and Contr. Nucl. Fusion Res. 1, 287, Novosibirsk 1968.

³ R. F. DE VRIES, Rijnhuizen Report 69-52, Thesis, Utrecht 1969.

⁴ W. SCHUURMAN, C. BOBELDIJK, and R. F. DE VRIES, Plasma Phys. 11, 495 [1969].

⁵ R. F. DE VRIES et al., Proc. 3rd Eur. Conf. on Contr. Fusion and Plasma Phys., 88, Utrecht 1969.

⁶ E. M. LITTLE, W. E. QUINN, and F. L. RIBE, Phys. Fluids 4, 711 [1961].

⁷ H. A. B. BODIN et al., Nucl. Fusion Suppl. Pt. 2, 521 [1962].

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